Optimal Portfolio Choice in Real Terms: Measuring the Benefits of TIPS*

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Abstract

In this paper, we solve an optimal portfolio choice problem to measure the benefits of Treasury Inflation Indexed Securities (TIPS) to investors concerned with maximizing real wealth. We show how the introduction of a real riskless asset completes the investor asset space, by contrasting optimal portfolio allocations with and without such assets. We use historical data to quantify gains from availability of TIPS in the presence of other asset classes such as equities, commodities, and real estate. We draw a distinction between buy-and-hold long-term investors for whom TIPS fully displace nominal riskfree assets and short-term investors for whom TIPS improve the investment opportunity set of real returns. Finally, we show how gains from TIPS are tempered by availability of alternative assets that covary with inflation, such as gold and real estate.

Keywords: Portfolio Choice in Real Terms; Treasury Inflation Indexed Securities (TIPS); Buy-and-hold Long-term Investors; Money Illusion.

JEL classification: G11.

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1 Introduction

Over the few last decades, a large number of articles by academics and practitioners has examined the arguments for and against issuing inflation indexed securities. Treasury Inflation-Protected Securities (TIPS) are a particular type of indexed bonds which are issued by the U.S. Treasury who introduced them to the market in January 1997. Despite the fact that issuance of TIPS with different maturities has followed suit, there is no consensus among policymakers and academics on what particular benefits TIPS provide to the different stakeholders in the economy, see Dudley et al. (2009) and Fleckenstein et al. (2010).

In general, it is argued that inflation protected government bonds provide benefits to the Treasury, policymakers, and investors, see for example Shen (1995), Barr and Campbell (1997), and Deacon et al. (2004). From the Treasury's point of view, the main benefit of issuing this type of bond is that they may reduce borrowing costs by not having to pay the inflation risk premium. From the policymakers perspective, it is argued that by introducing inflation linked bonds, they can improve market information mechanisms and enhance the credibility of the monetary policy because their issuance incentivizes the government to take an active role in controlling inflation. Finally, from the investors' point of view, inflation-indexed bonds can protect lenders against the erosion of their purchasing power.

The main question we address in this paper is what benefits do TIPS provide to investors. To better understand and measure these benefits we focus on the different attributes of investors and on the investment opportunities they may have. We differentiate the types of investors according to their investment horizon, short-term and long-term, and according to their degree of risk aversion. Then, we measure the incremental value that TIPS provide to different types of investor in the presence of various combinations of asset classes: equity, commodities, and real estate.

We consider long-term and short-term investors because the time horizon over which the investor plans to hold TIPS is relevant for only if these are held until maturity do they offer full protection against inflation.[1] TIPS have been issued with maturities of 5, 10, 20, and 30 years which makes them the riskless asset in real terms (a perfect hedge against inflation) for buy-and-hold long-term investors whose investment horizon perfectly matches the maturity of a TIPS. On the other hand, short-term investors see TIPS as "risky" assets, both in nominal and real terms because changes in expected real rates affect TIPS' returns. Moreover, when real interest rates rise investors who purchased TIPS will suffer a capital loss in greater proportion than those who purchased conventional bonds with

[1] There is some minimal inflation basis risk included in TIPS due to the fact that: a) the investor's basket might differ from the basket used to calculate the CPI-U to which the TIPS is indexed; b) there is a three month lag in the indexation rule; c) there are tax considerations; and, d) there is reinvestment risk arising from the coupon flows received before maturity.

the same maturity.[2] At the same time however, short-term investors may benefit from the introduction of TIPS if they can improve the investment efficient frontier to increase the returns per unit of risk.

This paper contributes to the literature in two respects. First, we solve an optimal portfolio choice problem in real terms to measure the benefits of TIPS from the investor's point of view. We assume that the investor's strategy consists of finding the optimal allocation over a fixed horizon without rebalancing at intermediate points in time. One of the empirical issues that the model handles is the possible mismatch between the investor's horizon and the maturity of TIPS. Since the first issuance in 1997 until now, the maturity of most outstanding TIPS at any point in time has been more than one year. Moreover, off-the-run TIPS with less than one year to maturity are not easy to find in the secondary market which results in extremely high transaction costs. Thus, only buy-and-hold long-term investors have been able to lend at the riskfree rate in real terms. The model shows that short-term investors deal with uncertainty about inflation through the covariances between the nominal returns of risky assets (one of which is TIPS) and inflation. In general, from our analysis it follows that it is only useful to distinguish the nominal from the real optimal portfolio choice problem if there is uncertainty about future inflation rates and there is a riskless asset in real terms, or when there is uncertainty about future inflation rates and assets in the investment opportunity set covary with inflation.

Our second contribution is to measure the empirical benefits that TIPS provide to investors using market data that spans the period from the first issuance of TIPS in 1997 until 2010. We show that while long-term investors can take advantage of the diversification effects that TIPS provide, as well as serving as the safe asset in their long-term investment problem, short-term investors may find them useful in improving the investment opportunity set in real terms. We summarize some of our findings according to the investment horizon of the investor: short-term and long-term.

For short-term investors we highlight four empirical findings. First, we find that risk averse short-term investors who are not affected by money illusion find it optimal to replace part of their investment in long-term nominal bonds with TIPS for two reasons. One, TIPS yield a slightly higher average return than nominal bonds, and two, the covariance of TIPS' nominal returns with inflation is higher than the covariance of the returns of nominal bonds with inflation. Second, the positive correlation of TIPS' nominal returns with inflation makes TIPS desirable for highly risk averse investors since they can be used to reduce the portfolio variance in real terms. Third, although the relative benefits from the introduction of TIPS diminish when the short-term investor has a wider investment opportunity set which might include gold, commodities, or real estate, highly risk averse investors still devote a fraction of their wealth to TIPS. Interestingly, when commodities are available,

[2]TIPS have longer duration than nominal T-bonds with the same maturity. Broadly speaking, the duration of a bond is the length of time before the bond is due to be repaid. Thus, it is a measure of the of bond's price sensitivity to interest rate movements.

the improvement to highly risk averse investors decreases because commodities are a better hedge against inflation than TIPS. Finally, investors characterized by low levels of risk aversion do not obtain any benefit from the introduction of TIPS when there is a wider investment opportunity set that includes: stocks, nominal bonds, commodities, real estate, and the short-term nominal riskless asset (T-bill).

For buy-and-hold long-term investors we highlight four empirical findings. First, infinitely risk averse investors who are not affected by money illusion allocate all their wealth to the riskfree asset in real terms, as predicted by the theoretical model of Wachter (2003). Second, for all levels of relative risk aversion, nominal bonds are crowded out by TIPS. Third, when real estate is part of the investment opportunity set, the relative benefits from TIPS diminish because real estate's expected real return, corrected by risk, is high enough to outperform the real yield of TIPS. Finally, investors characterized by a log utility function do not obtain any benefits from the introduction of TIPS.

The remainder of this paper is structured as follows. Section 2 looks at the existing literature on TIPS and summarizes previous findings on the potential benefits of TIPS for different types of investors. Section 3 derives the model that we employ to analyze the benefits of TIPS. Section 4 presents TIPS data and discusses its statistical properties since their first issuance in 1997 until 2010. Section 5 presents the empirical results and Section 6 concludes.

2 Literature review

The literature on investment allocation and indexed bonds is too large to cover here, so we focus on articles that are directly relevant to the subject matter of our paper. We summarize the main findings of papers that have: discussed inflation as a variable which affects asset allocation; and studied the empirical benefits that TIPS provide to short-term investors.

The seminal work of Markowitz (1952) provides a mean-variance framework for asset allocation. This analysis has been followed by a large number of studies that have stressed different aspects of the portfolio allocation problem. Some of the extensions that appeared in the 1970s and 1980s introduce inflation as a relevant variable (see for example Sarnat (1973), Biger (1975), Lintner (1975), Friend et al. (1976), Solnik (1978), and Levy and Levy (1987)). The most significant result of these studies is that when there is uncertainty about future inflation the riskless asset should be a one-period inflation-linked bond.

More recent articles employ the mean-variance analysis to measure the empirical benefits of TIPS for short-term investors, e.g. Lucas and Quek (1998), Kopcke and Kimball (1999), Roll (2004), Kothari and Shanken (2004), Hunter and Simon (2005), Brière and Signori (2009), among others. The conclusions of these studies vary depending on: i) the number of assets considered in the investment opportunity set; ii) the investment horizon employed in the calculations; iii) amount of data employed; or iv) assumptions made to compute returns. Moreover, apart from Kothari and Shanken (2004), they all assume that investors make allocation decisions in nominal terms– investors are not worried about the purchasing power of their terminal wealth.

The early study of Kopcke and Kimball (1999), which uses only two years of TIPS data, finds that in periods of low and falling rates of inflation, short-term investors with any level of risk aversion decide not to invest in TIPS; and the optimal allocation consists of a mix of T-bills, conventional nominal bonds, and stocks. Furthermore, they find that the only scenario under which TIPS are included in the optimal portfolio is when investors are highly risk averse and they cannot invest in T-bills. In our paper we use 13 years of data and find that investors with any degree of risk aversion (apart from log utility) will include TIPS in the optimal portfolio which also contains T-bills, conventional nominal bonds, and stocks.

Hunter and Simon (2005) use conditional mean-variance spanning tests to provide evidence that TIPS do not provide statistically significant diversification benefits to short-term investors that hold cash, nominal bonds, and equities. In the same vein, Brière and Signori (2009) show that the combined effect of stable expectations of inflation rates and the increase in the liquidity of TIPS results in a decreasing ability of TIPS to provide diversification effects in the portfolio due to their high correlation with nominal bonds. These two studies assume that investors do not care about the purchasing power of their final wealth. However, in light of the results of our paper, the conclusions of these two papers could result from assuming that investors might suffer from money illusion and the fact that we employ approximately five additional years of data.

Among the studies which find that TIPS provide benefits to investors is that of Roll (2004) who looks at the correlations of TIPS' returns with the returns of nominal bonds and equity between January 1997 and September 2003. Roll finds that TIPS improve the investment efficient frontier for short-term investors; which is consistent with our empirical results depending on which asset classes are considered. Kothari and Shanken (2004) study the optimal portfolio implications when both real and nominal returns are considered. They conclude that in an efficient portfolio with a one-year investment horizon and with assets restricted to stocks and bonds, substantial weight should be given to indexed bonds. They also speculate that: "It will be interesting to see whether this conclusion persists when allocations over longer horizons and across a broader range of assets, including global equities and bonds, are examined in future research."

The academic literature recognizes that TIPS are the safest asset for buy-and-hold long-term investors, but to the best of our knowledge this is the first paper that investigates the empirical benefits of their introduction. Previous work has looked at the benefits from introducing in the investment opportunity set assets that yield a real return. For example, Campbell et al. (2003) show that an infinitely-lived investor with Epstein-Zin preferences greatly benefits from the addition of a (hypothetical) real perpetuity or consol to his investment opportunity set that includes nominal bonds and stocks.

Among the theoretical papers that study the role of bonds that protect the bearer against inflation are Campbell and Viceira (2001), Campbell et al. (2003), Brennan and Xia (2002), Illeditsch (2009) and Wachter (2003). Campbell and Viceira (2001) and Campbell et al. (2003) show that in a world where investment opportunities are time-varying, an inflation-indexed perpetuity bond is the riskless asset for infinite-lived investors who care about the stream of consumption in every period. Similarly, Brennan and Xia develop an optimal dynamic portfolio problem for a finite-lived investor who is able to invest in stocks or nominal bonds. They show that an infinitely risk averse investor, who is unconstrained to take short positions, invests in a mix of nominal bonds to replicate the return of an inflation indexed bond with maturity equal to the remaining investment horizon. Illeditsch (2009) extends previous work and includes inflation protected bonds in the analysis. He finds that the real instantaneously risk-free asset can be obtained with a long position in inflation-protected bonds, and a zero-investment portfolio of nominal bonds together with the nominal money market account. Finally, Wachter formalizes the "preferred habitat" hypothesis of Modigliani and Sutch (1966) and shows that investors who keep their investment profile fixed for a known length of time, will consider a bond with maturity equal to their investment period as the riskless asset.

3 The Model: One-Period Portfolio Choice with Inflation

The classical mean-variance analysis for portfolio selection is usually posed in nominal terms, and if inflation is considered, the general approach implicitly assumes one of the following: i) investors suffer from money illusion; ii) the conditional variance of the inflation rate is zero; iii) there is no riskless real asset and assets' nominal returns are uncorrelated with inflation.

These three assumptions may not hold in practice and will adversely affect the portfolio allocation of investors that wish to protect their wealth from inflation. First, although investors may suffer from money illusion,[3] the potential benefits stemming from the introduction of a new asset which may be correlated with inflation should be measured in a framework stated in real terms. Otherwise the benefits may be under or overestimated. Second, certainty about future inflation rates may hold when the investment horizon is very short, but it is untenable for a long investment horizon. Finally, the introduction of TIPS allows buy-and-hold long-term investors to lend at a real riskless rate and empirical evidence rejects the assumption of independence between inflation and assets' nominal returns.[4]

[3] Cohen et al. (2005) provide empirical evidence to support that the stock market suffers from money illusion.

[4] Several articles in the 70s report evidence of negative correlation between nominal stock returns and inflation for short-term horizons; Bodie (1976), Nelson (1976), Fama and Schwert (1977), Jaffe and Mandelker (1976), among others. At long-horizons, Boudoukh and Richardson (1993) find that nominal stock

In this section, we solve an optimal portfolio choice problem for investors who consider the effects of inflation in their investment holdings. Intuitively, investors can avoid exposure to inflation risk by investing in a riskless asset in real terms and/or by investing in assets that covary with inflation. Therefore, we solve the investment allocation problem for investors both with and without a riskfree asset in real terms.

3.1 Portfolio choice with a riskless real asset

We consider the optimal investment allocation of investors who are not worried about what may happen beyond the immediate next period and care about the purchasing power of their wealth.[5] We assume that investors have a power utility function, defined over terminal real wealth and characterized by the Arrow-Pratt relative risk aversion coefficient γ. The investor's maximization problem is

$$\max \quad \mathrm{E}_t\left[\frac{W_{t+1}^{1-\gamma}}{(1-\gamma)}\right] \tag{1}$$

subject to the budget constraint

$$W_{t+1} = \big(1 + R_{p,t+1}\big)W_t\,. \tag{2}$$

The terminal real wealth W_{t+1} is equal to the initial real wealth W_t invested in the portfolio plus the portfolio's real return $R_{p,t+1}$. Under the assumption that the portfolio's real return is lognormally distributed and with a log transformation of the power utility function we re-write the maximization problem as

$$\max \quad (1-\gamma)\mathrm{E}_t\big[w_{t+1}\big] + \frac{1}{2}(1-\gamma)^2\sigma^2_{w,t} \tag{3}$$

$$\text{s.t.} \quad w_{t+1} = r_{p,t+1} + w_t, \tag{4}$$

where $w_t = \ln\big(W_t\big)$ is the log of the real wealth; $r_{p,t+1} = \ln\big(1 + R_{p,t+1}\big)$ is the log of the portfolio's real return;[6] and,

$$\mathrm{E}_t\big[w_{t+1}\big] = \mathrm{E}_t\big[r_{p,t+1}\big] + w_t \qquad \text{and} \qquad \mathrm{Var}_t\big[w_{t+1}\big] = \sigma^2_{w,t} = \sigma^2_{r_p,t} \tag{5}$$

are the expected value and variance of the log of real wealth, respectively. Dividing equation (3) by $(1-\gamma)$ and considering (4) and (5), the investor's problem becomes

$$\max \quad \mathrm{E}_t\big[r_{p,t+1}\big] + \frac{1}{2}(1-\gamma)\sigma^2_{r_p,t}\,. \tag{6}$$

returns are positively related (ex-ante and ex-post) to inflation at longer horizons. Roache and Attie (2009) provide a detailed literature review about the properties of different asset to hedge inflation.

[5] We assume that all investors are price takers, and that there are no taxes or transaction costs.

[6] For simplicity, we label it in this way even when it is the log of one plus the portfolio's real return.

Now, if the investor is affected by money illusion then the optimal allocation problem becomes[7]

$$\max \quad \mathrm{E}_t\big[r^{\$}_{p,t+1}\big] + \frac{1}{2}(1-\gamma)\sigma^2_{r^{\$}_p,t}\,, \tag{7}$$

where $r^{\$}_{p,t+1} = \ln\big(1 + R^{\$}_{p,t+1}\big)$ is the log of the portfolio's nominal return; $\mathrm{E}_t\big[r^{\$}_{p,t+1}\big]$ and $\sigma^2_{r^{\$}_p,t}$ are its expected value and variance, respectively. Moreover, the log of the portfolio's real return is given by

$$r_{p,t+1} = r^{\$}_{p,t+1} - \pi_{t+1}, \tag{8}$$

where $\pi_{t+1} = \ln\big(1+\Pi_{t+1}\big)$ and Π_{t+1} is the $t+1$ inflation rate. In order to simplify notation, we omit the time subindex from this point onwards. The problem for an investor who is not affected by money illusion can be re-written as

$$\max \quad \mathrm{E}\big[r^{\$}_p\big] - \mathrm{E}\big[\pi\big] + \frac{1}{2}(1-\gamma)\big(\sigma^2_{r^{\$}_p} + \sigma^2_{\pi} - 2\sigma_{r^{\$}_p\pi}\big). \tag{9}$$

The difference between the maximization problem solved for investors who are not affected by money illusion, equation (9), and for those who are, equation (7), is given by the variance of inflation and its correlation with financial assets, since the expected inflation rate $\mathrm{E}\big[\pi\big]$ cannot be affected by investors. In the particular case where inflation rates are conditionally certain ($\sigma^2_\pi = \sigma_{r^{\$}_p\pi} = 0$), the optimal portfolio under real and nominal terms is the same.

Note that in equation (9), the term $\frac{1}{2}\big(\sigma^2_{r^{\$}_p} + \sigma^2_\pi - 2\sigma_{r^{\$}_p\pi}\big)$ is Jensen's correction resulting from working in logs; while $-\frac{\gamma}{2}\big(\sigma^2_{r^{\$}_p} + \sigma^2_\pi - 2\sigma_{r^{\$}_p\pi}\big)$ is the investor's risk aversion adjustment. There are two particular cases where the maximization problem (9) is simplified. First, when the investor has logarithmic utility, characterized by $\gamma = 1$, the maximization problem in real terms is equivalent to the maximization problem in nominal terms; Jensen's correction and the investor's risk aversion adjustment cancel out. Thus, when there is uncertainty about inflation rates and investors' preferences are not characterized by a log utility function, money illusion produces inefficient portfolio selections. Second, when the investor is risk-neutral, characterized by $\gamma = 0$, equation (9) only includes Jensen's correction term.

From equation (9), we see that the portfolio allocation problem depends on the log of the portfolio's nominal return, which is not equal to the linear combination of the log of the nominal returns of the assets in the portfolio. To address this issue, we employ the approach of Campbell and Viceira (2002) who propose a second-order Taylor approximation of the log portfolio nominal return,[8] and we assume that the benchmark asset is the

[7] See Campbell and Viceira (2002).

[8] Campbell and Viceira (2002) remark that short horizon effects appear when the approximation is applied over long holding periods. Gil-Bazo (2006) showed that the approximate solution performs remarkably well under the stationary assumption, even for a long investment horizon.

real riskfree rate

$$r_p^{\$} = r_0 + \pi + \alpha'\big(\mathbf{r}^{\$} - \pi\mathbf{1} - r_0\mathbf{1}\big) + \frac{1}{2}\alpha'\tilde{\sigma}^2 - \frac{1}{2}\alpha'\tilde{\Sigma}\alpha\,. \tag{10}$$

Here $\mathbf{r}^{\$}$ is an $n \times 1$ vector of assets' log nominal returns; r_0 is the log real riskfree rate; α is a column vector with the allocation of the n risky assets in real terms; $\mathbf{1}$ is an $n \times 1$ column vector of ones;

$$\tilde{\Sigma} = \Sigma + \sigma_\pi^2 J - \sigma_{(r^{\$}\pi)}\mathbf{1}' - \mathbf{1}\sigma'_{(r^{\$}\pi)} \tag{11}$$

is the covariance matrix of real returns of the risky assets where Σ is the covariance matrix of nominal returns of the risky assets; J is an $n \times n$ matrix of ones; and $\sigma_{(r^{\$}\pi)}$ is a column vector with covariances between nominal returns and inflation. The diagonal elements of $\tilde{\Sigma}$ are the variances of the real return of asset i given by

$$\tilde{\sigma}_i^2 = \sigma_{r_i^{\$}}^2 + \sigma_\pi^2 - 2\sigma_{(r_i^{\$}\pi)}\,, \tag{12}$$

and the off-diagonal elements of $\tilde{\Sigma}$ are the covariances between the real rate of return of the risky assets i and j:

$$\tilde{\sigma}_{ij} = \sigma_{r_i^{\$}, r_j^{\$}} + \sigma_\pi^2 - \sigma_{(r_i^{\$}\pi)} - \sigma_{(r_j^{\$}\pi)}\,. \tag{13}$$

We use (10) to calculate

$$\mathrm{E}\big[r_p^{\$}\big] = (1 - \alpha'\mathbf{1})r_0 + \mathrm{E}\big[\pi\big] + \alpha'\mathrm{E}\big[\mathbf{r}^{\$} - \pi\mathbf{1}\big] + \frac{1}{2}\alpha'\tilde{\sigma}^2 - \frac{1}{2}\alpha'\tilde{\Sigma}\alpha\,, \tag{14}$$

$$\mathrm{Var}\big[r_p^{\$}\big] = \sigma_\pi^2 + \alpha'\tilde{\Sigma}\alpha + 2\alpha'\sigma_{(r^{\$}\pi)} - 2\alpha'\sigma_\pi^2\mathbf{1}\,, \tag{15}$$

$$\mathrm{Cov}\big[r_p^{\$}, \pi\big] = \sigma_{r_p^{\$}\pi} = \sigma_\pi^2 + \alpha'\sigma_{(r^{\$}\pi)} - \alpha'\sigma_\pi^2\mathbf{1}\,. \tag{16}$$

Substituting (14), (15), and (16) into equation (9), lays out the one-period optimization problem for an investor in real terms with a riskless real asset[9]

$$\max_{\alpha} \quad \alpha'\mathrm{E}\big[\mathbf{r}^{\$} - \pi\mathbf{1} - r_0\mathbf{1}\big] + \frac{1}{2}\alpha'\tilde{\sigma}^2 - \frac{\gamma}{2}\alpha'\tilde{\Sigma}\alpha. \tag{17}$$

The solution of the maximization problem in (17) is

$$\tilde{\alpha}^* = \frac{1}{\gamma}\tilde{\Sigma}^{-1}\left(\mathrm{E}\big[\mathbf{r}^{\$} - r_0\mathbf{1} - \pi\mathbf{1}\big] + \frac{\tilde{\sigma}^2}{2}\right), \tag{18}$$

where the optimal allocation in the real riskfree assets is $1 - \mathbf{1}'\tilde{\alpha}^*$. Note that when the inflation rate is conditionally certain, $\tilde{\sigma}^2 = \sigma^2$ and $\tilde{\Sigma} = \Sigma$, the optimal solution coincides with that of the problem stated in nominal terms.

[9] The problem can easily be generalized for the case of buy-and-hold investors with longer investment horizons, see Campbell and Viceira (2004).

Imposing short-selling constraints in problem (17):

$$\tilde{\alpha}_i^* = 0 \qquad \Rightarrow \quad \mathrm{E}\big[r_i^\$ - \pi\big] + \frac{1}{2}\tilde{\sigma}^2_{r_i^\$} - \gamma \sum_{\substack{j=1 \\ j\neq i}}^{n-1} \tilde{\alpha}_j^* \tilde{\sigma}_{(i,j)} < r_0 \tag{19}$$

$$\mathbf{1}'\tilde{\alpha}^* = 1 \quad \Rightarrow \quad \mathrm{E}\big[r_i^\$ - \pi\big] + \frac{1}{2}\tilde{\sigma}^2_{r_i^\$} - \gamma \sum_{j=1}^{n} \tilde{\alpha}_j^* \tilde{\sigma}_{(i,j)} > r_0 \quad \text{for all } i : \tilde{\alpha}_i^* \geq 0. \tag{20}$$

Equation (19) shows the case when the short-selling restriction for a risky asset is binding. If the expected real return of asset i, corrected by risk, is lower than the real riskfree rate, the investor would like to short-sell it. However, since short-selling is not allowed, the investor sets his holdings in that particular asset to zero. Note that the second term in the left-hand side (LHS) corresponds to Jensen's correction of the log function, while the third one is the risk aversion correction.

Equation (20) restricts the investor from borrowing money at the real riskfree rate; investors are not able to issue inflation-linked bonds. The investor would like to borrow money at the real riskfree rate when the expected real return of assets, corrected by risk, is higher than the real riskfree rate. Investors will benefit from the introduction of the real riskless asset if they are able to borrow at that rate or if the real interest rate is high enough.

By combining equations (19) and (20), we observe that the investor is locally indifferent between investing in the nominal or the real riskfree asset when

$$r_0^\$ - r_0 + \frac{1}{2}\sigma_\pi^2 = \mathrm{E}\big[\pi\big] + \gamma \sum_{j=1}^{n-1} \tilde{\alpha}_j^* \big(\sigma_\pi^2 - \sigma_{(r_j^\$ \pi)}\big)\,, \tag{21}$$

since

$$\tilde{\sigma}^2_{r_0^\$} = \sigma_\pi^2 \qquad \text{and} \qquad \tilde{\sigma}_{(r_0^\$ r_j^\$)} = \sigma_\pi^2 - \sigma_{(r_j^\$ \pi)}\,.$$

Here $r_0^\$$ is the log nominal riskfree rate, which is considered a risky asset by investors who are not affected by money illusion. The right-hand side of (21) is the one-period break-even inflation rate (BEI),[10] which consists of two terms: a) the expected inflation; and, b) the inflation risk premium that depends not only on the degree of risk aversion, but also on the volatility of inflation and the covariances between the nominal returns of the risky assets with inflation. Thus, the inflation risk premium cannot be analyzed by solely considering the riskfree assets in nominal and real terms; all assets in the economy, specially those that are potential hedgers against inflation, will affect the inflation risk premium.[11]

[10] Academics and practitioners refer to the break-even inflation rates as the difference between nominal and real bond's yield of the same maturity.

[11] Note that equation (21) with $\gamma = 0$ states the Fisher equation in a risk-neutral world. The log nominal riskfree rate is equal to the sum of the log real riskfree rate and the expected rate of inflation.

3.2 Portfolio choice without a riskless real asset

If investors are not able to find a riskless real asset, then the second-order Taylor approximation of the logarithm of the nominal portfolio return is

$$r_p^{\$} = r_0^{\$} + \alpha'\big(\mathbf{r}^{\$} - r_0^{\$}\mathbf{1}\big) + \frac{1}{2}\alpha'\sigma^2 - \frac{1}{2}\alpha'\Sigma\alpha\,. \tag{22}$$

Here Σ is the covariance matrix of the returns of the risky assets in nominal terms, and σ^2 represents its diagonal elements. The expected value, variance, and covariance terms of equation (22) are:

$$\mathrm{E}\big[r_p^{\$}\big] = (1-\alpha'\mathbf{1})r_0^{\$} + \alpha'\mathrm{E}\big[\mathbf{r}^{\$}\big] + \frac{1}{2}\alpha'\sigma^2 - \frac{1}{2}\alpha'\Sigma\alpha\,, \tag{23}$$

$$\mathrm{Var}\big[r_p^{\$}\big] = \alpha'\Sigma\alpha\,, \tag{24}$$

$$\mathrm{Cov}\big[r_p^{\$},\pi\big] = \sigma_{r_p^{\$}\pi} = \alpha'\sigma_{(\mathbf{r}^{\$}\pi)}\,. \tag{25}$$

Substituting (23), (24), and (25), into (9) results in the myopic optimization problem for an investor in real terms without a riskless real asset:

$$\max_{\alpha} \quad \alpha'\mathrm{E}\big[\mathbf{r}^{\$} - r_0^{\$}\mathbf{1}\big] + \frac{1}{2}\alpha'\sigma^2 - \frac{\gamma}{2}\alpha'\Sigma\alpha - (1-\gamma)\alpha'\sigma_{(\mathbf{r}^{\$}\pi)}\,, \tag{26}$$

with solution

$$\alpha^* = \frac{1}{\gamma}\Sigma^{-1}\left(\mathrm{E}\big[\mathbf{r}^{\$} - r_0^{\$}\mathbf{1}\big] + \frac{\sigma^2}{2} + (\gamma-1)\sigma_{(\mathbf{r}^{\$}\pi)}\right). \tag{27}$$

In the absence of a real riskfree asset, investors deal with uncertainty about inflation through the covariances between the nominal returns of risky assets and inflation. Securities which are correlated with inflation help to hedge against inflation risk, reducing the portfolio variance in real terms. When risky assets are uncorrelated with inflation or when investors are characterized by a log utility function, the optimal solution will be the same as in the nominal case.[12] Imposing short-selling constraints:

$$\alpha_i^* = 0 \qquad \Rightarrow \qquad \mathrm{E}\big[r_i^{\$}\big] + \frac{1}{2}\big(\sigma^2_{r_i^{\$}} - 2\sigma_{(r_i^{\$}\pi)}\big) - \gamma\left(\sum_{\substack{j=1\\ j\neq i}}^{n-1}\alpha_j^*\sigma_{(i,j)} - \sigma_{(r_i^{\$}\pi)}\right) < r_0^{\$} \tag{28}$$

$$\mathbf{1}'\alpha^* = 1 \quad \Rightarrow \quad \mathrm{E}\big[r_i^{\$}\big] + \frac{1}{2}\big(\sigma^2_{r_i^{\$}} - 2\sigma_{(r_i^{\$}\pi)}\big) - \gamma\sum_{j=1}^{n}\alpha_j^*\big(\sigma_{(i,j)} - \sigma_{(r_i^{\$}\pi)}\big) > r_0^{\$}, \quad \text{for all } i:\alpha_i^*\geq 0. \tag{29}$$

Equation (28) shows the case when the short-selling restriction for a risky asset is binding. If the expected nominal return of asset i, corrected by risk, is lower than the nominal riskfree rate, the investor sets his holdings in that particular asset to zero. Note that the

[12] Levy and Levy (1987) show that when there is no riskless real asset, and inflation and nominal returns are independent, the real and nominal efficient sets are identical.

second term in the LHS of the inequality corresponds to Jensen's correction, while the third corresponds to the risk aversion correction.

Equation (29) restricts the investor from borrowing money at the nominal riskfree rate. The investor would like to borrow money at the nominal riskfree rate when the expected nominal return of assets, corrected by risk, is higher than the nominal riskfree rate.

We can gain further insight into how investors choose their optimal allocations by re-arranging equation (28) as

$$\alpha_i^* = 0 \qquad \Rightarrow \qquad \mathrm{E}\big[r_i^\$\big] + \frac{1}{2}\sigma^2_{r_i^\$} - \gamma \sum_{\substack{j=1 \\ j\neq i}}^{n-1} \alpha_j^* \sigma_{(i,j)} + \sigma_{(r_i^\$ \pi)}\big(\gamma - 1\big) < r_0^\$ \tag{30}$$

to observe the effect of the covariance between nominal returns of risky assets and inflation. Compared to an investor who cares about inflation, an investor affected by money illusion and with a coefficient of relative risk aversion greater than one, will demand a higher (lower) nominal expected return if the nominal return of the risky asset covaries positively (negatively) with inflation.

4 Treasury Inflation-Protected Securities (TIPS)

The primary feature of TIPS is that their principal is indexed to the U.S. non-seasonally adjusted consumer price index for all urban consumers (CPI-U NSA). Then, if an investor holds TIPS until maturity, he will receive a known return in real terms for his investment. On the other hand, before maturity, TIPS' returns are uncertain both in real and nominal terms.

TIPS have been issued with maturities of 5, 10, 20, and 30 years. The 5-year TIPS were first issued toward the end of 1997, but TIPS with this maturity were discontinued until the end of 2004 when the Treasury started yearly issues. The 10-year class of TIPS is the only one which has been continuously issued. Initially, the 10-year TIPS were issued once a year, but from July 2003 they have been issued twice a year. From 1998 until 2001 the Treasury issued three lots of 30-year TIPS and were discontinued until 2010. Between 2005 and 2009 there were five yearly 20-year emissions.

Figure 1 shows a time series of outstanding TIPS grouped according to their term to maturity. The lack of outstanding TIPS with maturity less than 1-year between 1997 and 2010 reflects the absence of a riskless asset in real terms for short-term investors. Although since 2005 there were more opportunities to find outstanding (off-the-run) TIPS with shorter maturities, they were primarily issued to provide a safe asset for investors with long investment horizon. Hence, while buy-and-hold long-term investors have been able to invest, but not borrow, at the real riskfree rate, short-term investors have had no access to a riskless asset in real terms and see TIPS as "risky" assets.

4.1 Data

We create a data set of nominal monthly returns for all TIPS issued before August 2009. A comparable nominal Treasury bond is selected as a benchmark for each TIPS, according to the issue and maturity date. Monthly TIPS returns are calculated using accrued interest, capital gains, and inflation adjustments. The period of study spans from March 1997 to March 2010 which results in 157 monthly observations. The period encompasses an entire U.S. business cycle and two recessions.[13]

We group TIPS and nominal bonds according to their maturity: short-term (ST) notes (4 to 5 years), medium term (MT) notes (9 to 10 years) and long-term (LT) bonds (more than 10 years). See Table 1 and Table 2 for descriptive statistics. We construct bond indices that represent the performance of each group of Treasury bonds. For TIPS indices, the reference security is the newest TIPS issuance within the group. For Nominal indices, we obtain a comparable nominal bond closest in maturity to the correspondent TIPS.

The rest of the data set is composed of alternative investment assets and a measure of inflation. For the different asset classes, we use a representative index: for equity we use the S&P500 index; for commodities we employ gold prices and the S&P GSCI index; and for real estate we use the S&P/Case-Shiller Home Price index. Inflation is measured as the log-difference of the CPI-U NSA for two consecutive months.

Returns. Table 3 exhibits nominal statistics of log returns for bond indices and the rest of the financial assets for three sample periods: Panel A: the entire period from March 1997 to March 2010; Panel B: the whole business cycle (from peak to peak) according to the Business Cycle Committee of the National Bureau of Economic Research, from March 2001 to December 2007; and, Panel C: the last two U.S. recessions, from March 2001 to November 2001 and from December 2007 to June 2009. Means, medians and standard deviations of nominal monthly log-returns are annualized.

Panel A in Table 3 shows that longer term TIPS experienced higher average returns than the shorter term maturities, suggesting the presence of a term premium in TIPS' nominal returns. The long-term TIPS index had a mean monthly return[14] of 8.18% per year for the entire sample, while 5 and 10-year TIPS indices experienced a mean return of 6.31% and 6.49%, respectively. At the same time, TIPS have outperformed comparable nominal bonds for the complete period. Panel B corroborates the presence of a bond term premium in TIPS during the entire U.S. Business Cycle from March 2001 to December 2007. Panel B also shows that the outperformance of TIPS over nominal bonds is even higher during this sub-period.

Interestingly, during the last two U.S. recessions (March 2001 to November 2001 and

[13] http://www.nber.org/cycles/cyclesmain.html. All market price data are taken from Bloomberg.
[14] Average returns are equal to mean log-returns plus Jensen's correction, $\frac{\sigma^2}{2}$.

December 2007 to June 2009) nominal bonds outperformed TIPS' nominal returns and short-term bonds showed higher realized nominal log-returns than long-term bonds, see Panel C in Table 3. The average inflation rates of the recession sub-sample was 1.20%, lower than the average observed for the entire business cycle period of 2.62%. During recessions one would expect short-term real interest rates to go down creating positive returns for bonds. Besides, in a framework of a stable Phillips curve inflation rates also will drop. Under this context, nominal bonds would be preferable during recessions than TIPS with similar maturity.

Real vs nominal yields. The introduction of TIPS allows long-term investors to invest in a risk-free asset that guarantees a real return. The long-term excess returns in real terms for the 10-year nominal bonds over the 10-year TIPS is -0.31 for the complete period. The general agreement about the negative premium is that it reflects the lower liquidity of TIPS relative to comparable nominal bonds.[15] A liquidity premium in TIPS over comparable nominal bonds makes sense when buyers of TIPS consider the chance to unwind the position before maturity. However, if all individuals in the economy were buy-and-hold long-term investors, the liquidity premium would be zero, and a negative long-term real excess return for nominal bonds would imply a negative inflation risk premium, which seems odd in a risk-averse world where individuals are not affected by money illusion.

Volatility. The volatility of the nominal returns of TIPS is lower than the volatility of the returns of comparable nominal bonds when the entire period is considered. During the business cycle, TIPS' nominal returns also exhibit lower volatility than nominal bonds, except for long-term bonds.

As it was documented by Brière and Signori (2009), and Campbell et al. (2009), TIPS' volatility has increased relative to that of nominal bonds since 2003. Strikingly, during the financial crisis that started in 2008 the volatility of TIPS was equal or even higher than that of nominal bonds. TIPS' nominal returns are expected to have higher variance than the variance of the returns of nominal bonds when: the variance of realized inflation is higher than the variance of expectations about future inflation rates; or, the variance of any further premia included in TIPS is higher than the variance of any premia included in nominal bonds. Realized inflation exhibits low and almost constant volatility with an increase in the last part of 2008 due to high deflation rates. At the same time, the large changes in break-even inflation rates driven by an increase in TIPS' real yields (Figure 2) in the same period suggests that both effects have pushed the variance of TIPS' return, measured in nominal returns, above the variance of the returns of nominal bonds (Figure 3).

Correlations. Table 4 shows the correlation matrix of the set of financial assets for the entire period (Panel A) and for the whole business cycle (Panel B). As expected, TIPS' nominal returns are highly correlated among them because they depend on the same

[15] See Sack and Elsasser (2004), D'Amico et al. (2009), Pflueger and Viceira (2010), among others.

factors; fluctuations in the real interest rate, changes in any premium contained in TIPS, and realized inflation. Besides, TIPS with adjacent maturities and with longer durations have higher correlations among them. Both panels in Table 4 exhibit the high correlation between TIPS and nominal bonds, particularly for bonds with longer maturities. The minimum correlation among Treasury bonds for the entire period is observed between the 5-year Nominal index and the TIPS LT Bond index, 0.583, while the maximum correlation of 0.787 is between LT Bond.

An interesting point in both panels is the low correlation between realized inflation, measured by the CPI-U NSA, and TIPS' nominal returns. The longer the maturity of the TIPS, the lower the correlation between TIPS and realized inflation. In Panel A, the correlation of the TIPS 5-year index, 10-year index, and Bond index with inflation are 0.220, 0.174, and 0.094, respectively. If we consider data for the entire business cycle (Panel B), the correlation between TIPS' nominal returns and inflation is even lower. It is clear that realized inflation is not the predominant source of monthly nominal TIPS returns which may suggest that they do not provide a good hedge against inflation over short periods. Commodities, rather than gold, seem to be a better instrument to protect investors against inflation.

Table 4 shows that the correlation between the stock market and nominal bonds is lower than the correlation between the stock market and TIPS. Figure 4 plots a 3-year rolling beta of the TIPS and nominal 10-year indices.[16] Nominal bonds seem to offer a better hedge against the equity market than TIPS, at least over short horizons. Under the CAPM framework this would imply a positive risk premium contained in TIPS relative to nominal bonds.

5 Measuring the benefits of TIPS

In this section we compute and analyze the benefits that TIPS provide to investors. Our empirical study focuses on two aspects. First, we consider different aspects about investors' preferences: investment horizon, risk aversion, and attitude towards inflation. Second, we study how TIPS perform as a marginal security in the presence of other investment opportunities: nominal bonds, equity, commodities, gold, and real estate.[17] To illustrate the marginal benefit of TIPS, we compute the difference of the return of the optimal portfolio which includes TIPS; and the risk-adjusted expected return of the optimal portfolio, excluding TIPS;

$$\text{Benefits} = \mathrm{E}_t\left[r_{p,t+1}^{(tips)}\right] - \frac{\sigma_{r_p^{(tips)}}}{\sigma_{r_p}} \mathrm{E}_t\left[r_{p,t+1}\right]. \tag{31}$$

[16]The beta is calculated as the covariance between the bonds returns and the S&P 500 returns divided by the variance of the S&P 500 returns.

[17]Here we make the assumption that there is a real estate index that investors can purchase and liquidate with no transaction costs.

As discussed above, when the horizon over which investors hold assets does not match the maturity of any of the outstanding TIPS, these inflation protected bonds are risky and are considered by the investor as any other asset with uncertain payoff. On the other hand, if the horizon of investors and the maturity of TIPS coincide, then investors who maximize real wealth consider TIPS as the risk-free asset. Since most of the outstanding TIPS have maturities of more than one year, we regard those who have an investment horizon of one month and cannot find TIPS with this maturity short-term investors in our empirical study. Likewise, we regard those with a horizon of 10 years as buy-and-hold long-term investors because 10-year TIPS are the only ones who have been continuously issued since 1997.[18] Furthermore, throughout our analysis we assume that investors make decisions based on real terms; that is, investors do not suffer from money illusion.

Specifically, we use the one-period portfolio choice framework in real terms, developed in Section 3, to measure the benefits that TIPS provide to both short- and long-term investors. We consider a range of values for risk aversion and measure the marginal benefit of TIPS in the presence of different asset classes. In particular, for short-term investors we consider the combination of stocks, nominal bonds, commodities, real estate, and T-bills. Here T-bills represent the short-term riskless asset in nominal terms. For long-term investors we consider the same cases, but we exclude T-bills from all scenarios (because this would require the investor to rebalance her portfolio upon the expiry of the T-Bill) and instead include nominal bonds with the same maturity as the TIPS.

5.1 Short-term investors

We analyze the problem of short-term investors who maximize real wealth and are not able to invest in a riskless asset in real terms. In this case, the problem solved by investors is given by (26) and the optimal allocation is given in equation (27). In the absence of a riskless asset in real terms, the only way in which short-term investors can deal with uncertainty about inflation is through the covariances between the nominal returns of risky assets and inflation. For example, infinitely risk averse investors who are not affected by money illusion and are not allowed to short-sell assets, will invest a fraction of their wealth in risky assets which are positively correlated with inflation in order to reduce the portfolio variance in real terms. On the other hand, infinitely risk averse investors who maximize nominal wealth will allocate all their wealth to the nominal riskless asset.

Stocks, nominal bonds, and T-bills. To compare the marginal benefits of TIPS we use the case where short-term investors allocate their wealth to stocks, nominal bonds, and T-bills as a benchmark. Figure 5 shows the optimal weights, in percentage terms, of total invested wealth for different levels of relative risk aversion. The figure shows that for high (low) levels of risk-aversion, the investor relies heavily on T-Bills (stocks and 10-year

[18]Fleming and Krishnan (2009) provide evidence that trading activity in TIPS is concentrated in 10-year notes representing 71.6%.

nominal bonds) and that very little is invested in nominal 10-year bonds and stocks (T-bills). The absence of a riskless real asset, together with short-selling constraints, are able to explain "the asset allocation puzzle" stated by Canner et al. (1997); that is, aggressive investors hold a lower ratio of bonds to stocks than conservative investors.[19]

Stocks, 10-year nominal bonds, T-bills and 10-year TIPS. Figure 6 exhibits the optimal weights when TIPS are introduced. For all levels of relative risk aversion, except $\gamma = 1$ (log-utility), a significant part of investors' wealth is devoted to TIPS. Panel i in Table 5 shows that the optimal wealth devoted to TIPS varies from 4% for infinitely risk averse investors to 55% for investors with a degree of risk aversion of $\gamma = 10$. In the Table we show the optimal weight of each asset when TIPS are available and in parenthesis the optimal weight when TIPS are not available. As expected, TIPS seem to be a reasonable, but not perfect, substitute for nominal bonds with the same maturity. By looking at the difference between the optimal weights with and without TIPS we show the crowding out effects of TIPS. For instance for a $\gamma = 5$ the optimal weight devoted to long-term nominal bonds is half after introducing TIPS. Therefore, both classes of long-term bonds are included in the optimal portfolio of short-term investors. Extremely risk averse investors who maximize real wealth, invest a small fraction of their wealth into stocks (1.3%) and TIPS (3.7%) to hedge the inflation risk of T-bills.

In order to analyze the benefits of TIPS to investors, we calculate (31). Panel i in Table 5 reports the relative benefit of introducing TIPS for short-term investors over the period 1997 to 2010. For instance, while investors characterized by a $\gamma = 10$ obtain a benefits of $35bp$, extremely risk averse investors get almost $15bp$ by the introduction of TIPS. The positive correlation of TIPS' returns and inflation makes TIPS a useful instrument to reduce the inflation risk. On the other extreme, log-utility investors do not obtain any benefits from the introduction of TIPS.

If we restrict our data to cover the entire business cycle from March 2001 to December 2007, the benefits to investors from the introduction of TIPS are greater than those for the whole sample. Also, long-term nominal bonds are totally crowded out by long-term TIPS in the optimal portfolio of short-term investors. On the other hand, during recessionary periods (March 2001 to November 2001 and from December 2007 to June 2009) when average inflation rates are lower, long-term nominal bonds outperform long-term TIPS, the latter providing no benefits to short-term investors.

Stocks, nominal bonds, commodities, real estate, T-bills and TIPS. Panels ii-v in Table 5 show that when short-term investors have a wider investment opportunity set that might include gold, commodities or real estate, the relative benefits from the introduction of TIPS diminish, but still remain part of their optimal allocation. The

[19] Canner et al. (1997) document that financial advisors recommend that more risk averse investors should hold a higher ratio of bonds to stocks, which is inconsistent with the mutual-fund separation theorem. Under certain assumptions the theorem predicts that all investors should hold risky assets in the same proportion.

positive correlation of TIPS' nominal returns with inflation makes TIPS desirable for highly risk averse investors since it enables them to reduce the portfolio variance in real terms. Interestingly, when commodities are available, the improvement decreases because commodities are a better hedge than TIPS against inflation.[20] For example, for levels of risk aversion around $\gamma = 10$ we see that the introduction of TIPS produces benefits of $26bp$, see Panel iii in the same Table. Lastly, when all assets are considered, investors characterized by low levels of risk aversion, $\gamma < 5$, do not benefit from TIPS, while most risk averse investors with $5 < \gamma < \infty$ (approximately) do obtain benefits from TIPS. For instance, Panel v in Table 5 investors with a degree of risk aversion of $\gamma = 20$ devote almost 15% of their wealth to TIPS.

5.2 Buy-and-hold long-term investors

The introduction of TIPS provides buy-and-hold long-term investors with a riskfree asset in real terms. Here we employ equations (17) and (18) to compute the optimal weights and to measure the relative benefits that TIPS provide to buy-and-hold long-term investors who are not affected by money illusion. We assume that investors are neither allowed to borrow at the riskless rate in real terms, nor allowed to short-sell risky assets.

Stocks and 10-year nominal bonds. Figure 7 shows the optimal allocation weights when long-term investors can only purchase stocks and long-term nominal bonds with maturity 10 years which is the same as the time horizon over which the buy-and-hold investor solves the optimal allocation problem. Infinitely risk averse investors allocate a very small fraction of their wealth to stocks, instead investing all in the long-term nominal bonds. Whereas for investors with log-utility function all wealth is placed in stocks.

Stocks, 10-year nominal bonds, and 10-year TIPS. Figure 8 exhibits the optimal allocation of the investor's wealth after the introduction of TIPS. The introduction of TIPS allows long-term investors to buy an asset that guarantees a real return. The long-term excess returns in real terms for stock and nominal long-term bonds are 3.98% and -0.31%, respectively. Not surprisingly given the values of excess return in real terms, no weight is given to long-term nominal bonds which are totally crowded out by TIPS for all levels of relative risk aversion. As expected, the greatest benefits are observed for investors with higher levels of risk aversion, ranging from $21bp$ to $248bp$, see panel i in Table 6 . The salient point here is that buy-and-hold long-term investors will always replace long-term nominal bonds for long-term TIPS given historic real and nominal yields.

Stocks, 10-year nominal, commodities, real estate, and 10-year TIPS. Panels ii-v in Table 6 show the benefits of introducing TIPS as a marginal security when different asset classes are available. In general, the higher the relative risk aversion, the higher the benefits that investors enjoy regardless of how wide the investment opportunity

[20]See Table 4.

set is. As expected, infinitely risk averse investors who are not affected by money illusion allocate all their wealth to TIPS, as predicted by the theoretical model of Wachter (2003), obtaining the greatest benefits.

Our results show that when a wider set of asset classes is considered, the main benefits to long-term investors result from substituting long-term nominal bonds with TIPS. Panels iv and v in Table 6 show that when real estate investment are consider the benefits of investing in TIPS decrease for those investors who are not infinitely reluctant to bear risk. Intuitively, real estate investments are beneficial for long-term investors who are willing to tolerate some risk in their portfolio. For example, we see that if real estate is considered by investors with $\gamma < 5$, they will not purchase any TIPS because real estate's expected real return, corrected by risk, outperforms the real yield of TIPS. However, we note that the methodology to calculate the S&P/Case-Shiller index introduces smoothing effects that may underestimate the true volatility of real estate investments.[21] Finally, investors characterized by a log utility function do not obtain any benefits from the introduction of TIPS, regardless of the investment opportunities– indeed, they devote all their wealth to the asset with highest real expected return.

6 Concluding Remarks

In this paper we solve an optimal portfolio choice problem in real terms in order to measure what benefits do TIPS provide to investors. We show analytically that this approach should be used when there is uncertainty about future inflation rates and there is a riskless asset in real terms, or when there is uncertainty about future inflation rates and assets in the investment opportunity set covary with inflation. In other words, investors who are not affected by money illusion can deal about unexpected inflation rates through two possible channels: a) investing in the risk-free asset in real terms; and/or b) investing in those risky assets whose nominal returns covary with inflation.

TIPS have been issued with maturities of 5, 10, 20, and 30 years, thus they were primarily issued to provide a safe asset for investors with long investment horizons. The time horizon over which the investor plans to hold TIPS is relevant because only if these are held until maturity do they behave as the riskless asset in real terms. Therefore, to better understand and measure the benefits that TIPS provide to investors we differentiate the types of investors according to their investment horizon, short-term and buy-and-hold long-term, and according to their degree of risk aversion. We measure the historical benefits that TIPS provide to the two types of investor in the presence of different asset classes such as equity, commodities, and real estate.

We find that short-term risk averse investors who are not affected by money illusion find it optimal to replace part of their investment in long-term nominal bonds with TIPS

[21] We are grateful to Michael Brennan for underlying this point.

for two reasons. One, TIPS yield a slightly higher average return than nominal bonds, and two, the covariance of TIPS' nominal returns with inflation is higher than the covariance of the returns of nominal bonds with inflation. We also find that the positive correlation of TIPS' nominal returns with inflation makes TIPS desirable for highly risk averse short-term investors since they can be used to reduce the portfolio variance in real terms. Moreover, although the relative benefits from the introduction of TIPS diminish when the short-term investor has a wider investment opportunity set which might include gold, commodities, or real estate, highly risk averse short-term investors still devote a fraction of their wealth to TIPS. Interestingly, when commodities are available, the improvement to highly risk averse short-term investors decreases because commodities are a better hedge against inflation than TIPS. Finally, short-term investors characterized by low levels of risk aversion do not obtain any benefit from the introduction of TIPS when there is a wider investment opportunity set that includes: stocks, nominal bonds, commodities, real estate, and the short-term nominal riskless asset (T-bill).

For buy-and hold long-term investors we find that: infinitely risk averse investors who are not affected by money illusion allocate all their wealth to the riskfree asset in real terms, as predicted by the theoretical model of Wachter (2003); for all levels of relative risk aversion, nominal bonds are crowded out by TIPS; when real estate is part of the investment opportunity set, the relative benefits from TIPS diminish because real estate's expected real return, corrected by risk, is high enough to outperform the real yield of TIPS; and finally, investors characterized by a log utility function do not obtain any benefits from the introduction of TIPS.

The model can also be employed to analyze the one-period breakeven inflation rate (BEI). We show that by imposing short-selling constraints we obtain an expression for the BEI which consists of two terms: a) the expected inflation rate; and, b) the inflation risk premium which depends not only on the degree of risk aversion, but also on the volatility of inflation and the covariances between the nominal returns of the risky assets with inflation. Therefore, the inflation risk premium cannot be analyzed by solely considering the riskfree assets in nominal and real terms; all assets in the economy, specially those that are potential hedgers against inflation, will affect the inflation risk premium. This indicates that even in a risk averse world characterized by investors who are not affected by money illusion it is possible to observe a negative inflation risk premium. Moreover, when investors do not have access to a riskless asset in real terms the real yield of TIPS may include not only a liquidity premium but also a risk premium. Thus, two directions for future research are to better understand how the ivestment horizon of market participants affects the BEI of long-term bonds; and whether governments can save the inflation risk premium by issuing long-term inflation-indexed bonds.

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Table 1: Sample of TIPS and comparable Nominal Treasury bonds

We group TIPS and nominal bonds based on their maturity: short-term notes (4 to 5 years), medium term notes (9 to 10 years) and long-term bonds (more than 10 years). We obtain a comparable nominal bond by choosing the most recent on-the-run bond closest in maturity to our TIPS benchmark at the moment of the TIPS issuance.

	Date		Real					Date		Nominal			
	Issue	Maturity	Coupon %	Yield %	Price per $100	CUSIP		Issue	Maturity	Coupon %	Yield %	Price per $100	CUSIP
I) U.S. TIPS							**II) U.S. Treasury Nominal Bonds**						
a) Short-term notes (4-year to 5-year)							**a) Short-term notes (4-year to 5-year)**						
1.	15-Jul-97	15-Jul-02	3.625	3.744	99.462	9128273A8	1.	31-Jul-97	31-Jul-02	6.000	6.024	99.898	9128273C4
2.	29-Apr-05	15-Apr-10	0.875	1.200	99.624	912828CZ1	2.	15-Apr-05	15-Apr-10	4.000	4.046	99.794	912828DR8
3.	28-Apr-06	15-Apr-11	2.375	2.379	100.067	912828FB1	3.	1-May-06	30-Apr-11	4.875	4.964	99.610	912828FD7
4.	30-Apr-07	15-Apr-12	2.000	2.114	99.731	912828GN4	4.	30-Apr-07	30-Apr-12	4.500	4.579	99.651	912828GQ7
5.	30-Apr-08	15-Apr-13	0.625	0.745	99.561	912828HW3	5.	30-Apr-08	30-Apr-13	3.125	3.159	99.844	912828HY9
6.	30-Apr-09	15-Apr-14	1.250	1.278	100.113	912828KM1	6.	30-Apr-09	30-Apr-14	1.875	1.940	99.692	912828KN9
		mean	1.792	1.910					mean	4.063	4.119		
b) Notes (9-year to 10-year)							**b) Notes (9-year to 10-year)**						
1.	6-Feb-97	15-Jan-07	3.375	3.449	99.482	9128272M3	1.	18-Feb-97	15-Feb-07	6.250	6.374	99.092	9128272J0
2.	15-Jan-98	15-Jan-08	3.625	3.730	99.130	9128273T7	2.	17-Feb-98	15-Feb-08	5.500	5.558	99.559	9128273X8
3.	15-Jan-99	15-Jan-09	3.875	3.898	99.811	9128274Y5	3.	16-Feb-99	15-Nov-08	4.750	4.913	98.735	9128274V1
4.	18-Jan-00	15-Jan-10	4.250	4.338	99.298	9128275W8	4.	15-Feb-00	15-Feb-10	6.500	6.540	99.710	9128275Z1
5.	16-Jan-01	15-Jan-11	3.500	3.522	99.818	9128276R8	5.	15-Feb-01	15-Feb-11	5.000	5.067	99.479	9128276T4
6.	15-Jan-02	15-Jan-12	3.375	3.480	99.120	9128277J5	6.	15-Feb-02	15-Feb-12	4.875	4.880	99.961	9128277L0
7.	15-Jul-02	15-Jul-12	3.000	3.099	99.154	912828AF7	7.	15-Aug-02	15-Aug-12	4.375	4.390	99.880	912828AJ9
8.	15-Jul-03	15-Jul-13	1.875	1.999	98.881	912828BD1	8.	15-Aug-03	15-Aug-13	4.250	4.370	99.036	912828BH2
9.	15-Jan-04	15-Jan-14	2.000	2.019	99.829	912828BW9	9.	17-Feb-04	15-Feb-14	4.000	4.060	99.511	912828CA6
10.	15-Jul-04	15-Jul-14	2.000	2.020	99.820	912828CP3	10.	16-Aug-04	15-Aug-14	4.250	4.270	99.838	912828CT5
11.	18-Jan-05	15-Jan-15	1.625	1.725	99.091	912828DH0	11.	15-Feb-05	15-Feb-15	4.000	4.049	99.600	912828DM9
12.	15-Jul-05	15-Jul-15	1.875	1.939	99.421	912828EA4	12.	15-Aug-05	15-Aug-15	4.250	4.350	99.196	912828EE6
13.	17-Jan-06	15-Jan-16	2.000	2.025	99.723	912828ET3	13.	15-Feb-06	15-Feb-16	4.500	4.540	99.681	912828EW6
14.	17-Jul-06	15-Jul-16	2.500	2.550	99.593	912828FL9	14.	15-Aug-06	15-Aug-16	4.875	4.930	99.570	912828FQ8
15.	16-Jan-07	15-Jan-17	2.375	2.449	99.342	912828GD6	15.	15-Feb-07	15-Feb-17	4.625	4.740	99.093	912828GH7
16.	16-Jul-07	15-Jul-17	2.625	2.749	98.942	912828GX2	16.	15-Aug-07	15-Aug-17	4.750	4.855	99.176	912828HA1
17.	15-Jan-08	15-Jan-18	1.625	1.655	99.725	912828HN3	17.	15-Feb-08	15-Feb-18	3.500	3.620	99.001	912828HR4
18.	15-Jul-08	15-Jul-18	1.375	1.485	98.981	912828JE1	18.	15-Aug-08	15-Aug-18	4.000	4.075	99.389	912828JH4
19.	15-Jan-09	15-Jan-19	2.125	2.245	98.931	912828JX9	19.	17-Feb-09	15-Feb-19	2.750	2.818	99.411	912828KD1
20.	15-Jul-09	15-Jul-19	1.875	1.920	99.592	912828LA6	20.	17-Aug-09	15-Aug-19	3.625	3.734	99.097	912828LJ7
		mean	2.544	2.615					mean	4.531	4.607		
c) Bonds (19-year to 20-year and 29-year to 30-year)							**c) Bonds (19-year to 20-year and 29-year to 30-year)**						
1.	15-Apr-98	15-Apr-28	3.625	3.740	97.937	912810FD5	1.	17-Aug-98	15-Aug-28	5.500	5.590	98.697	912810FE3
2.	15-Apr-99	15-Apr-29	3.875	3.899	99.578	912810FH6	2.	16-Feb-99	15-Feb-29	5.250	5.298	99.282	912810FG8
3.	15-Oct-01	15-Apr-32	3.375	3.465	98.314	912810FQ6	3.	15-Aug-01	15-Feb-31	5.375	5.520	97.900	912810FP8
4.	31-Jan-05	15-Jan-25	2.375	2.000	107.552	912810FR4	4.	15-Feb-95	15-Feb-25	7.620	7.650	99.708	912810ET1
5.	31-Jan-06	15-Jan-26	2.000	2.039	98.949	912810FS2	5.	15-Feb-96	15-Feb-26	6.000	6.119	98.374	912810EW4
6.	31-Jan-07	15-Jan-27	2.375	2.420	99.213	912810PS1	6.	18-Feb-97	15-Feb-27	6.625	6.640	99.804	912810EZ7
7.	31-Jan-08	15-Jan-28	1.750	1.807	99.351	912810PV4	7.	17-Feb-98	15-Nov-27	6.125	5.822	104.238	912810FB9
8.	30-Jan-09	15-Jan-29	2.500	2.500	99.064	912810PZ5	8.	16-Feb-99	15-Feb-29	5.250	5.298	99.282	912810FG8
		mean	2.734	2.734					mean	5.968	5.992		

Table 2: Nominal monthly log returns of TIPS and comparable nominal Treasury bonds

We group TIPS and nominal bonds based on their maturity: short-term notes (4 to 5 years), medium term notes (9 to 10 years) and long-term bonds (more than 10 years). We obtain a comparable nominal bond by choosing the most recent on-the-run bond closest in maturity to our TIPS benchmark at the moment of the TIPS issuance.

Date	N	mean	med	monthly		std	skew	kurt	CUSIP
from/to				max	min				
I) U.S. TIPS									
a) Short-term notes (4-year to 5-year)									
Aug-97/Feb-02	55	6.18	6.26	1.36	-0.22	1.34	0.09	2.32	9128273A8
Nov-04/Mar-10	65	3.41	3.54	2.40	-4.58	3.43	-1.67	10.67	912828CZ1
May-06/Mar-10	47	5.22	5.61	3.20	-6.24	4.85	-2.09	12.55	912828FB1
May-07/Mar-10	35	5.84	6.51	3.83	-6.06	5.89	-1.45	7.74	912828GN4
May-08/Mar-10	23	3.35	3.72	2.56	-4.09	5.43	-0.87	3.93	912828HW3
May-09/Mar-10	11	6.96	7.75	2.16	-1.06	3.67	-0.14	1.73	912828KM1
b) Notes (9-year to 10-year)									
Mar-97/Dec-06	118	5.73	5.24	2.48	-1.80	2.71	0.18	3.70	9128272M3
Feb-98/Dec-07	119	6.13	5.38	2.94	-2.34	3.08	0.06	4.20	9128273T7
Feb-99/Dec-08	119	6.32	6.57	3.11	-2.82	3.42	-0.18	4.66	9128274Y5
Feb-00/Dec-09	119	6.48	6.97	3.07	-3.76	4.03	-0.70	5.28	9128275W8
Feb-01/Mar-10	110	6.03	6.28	3.69	-6.24	4.93	-1.21	7.51	9128276R8
Feb-02/Mar-10	98	6.44	6.62	3.81	-6.67	5.44	-1.17	7.35	9128277J5
Aug-02/Mar-10	92	5.92	5.43	4.08	-6.41	5.83	-0.86	5.97	912828AF7
Aug-03/Mar-10	80	5.89	5.65	4.15	-6.28	5.91	-0.97	5.91	912828BD1
Feb-04/Mar-10	74	5.34	5.21	4.22	-8.75	6.59	-1.56	9.47	912828BW9
Aug-04/Mar-10	68	5.50	4.82	4.47	-8.25	6.62	-1.12	8.14	912828CP3
Feb-05/Mar-10	62	4.97	5.22	5.32	-10.31	7.52	-1.60	11.33	912828DH0
Aug-05/Mar-10	56	5.46	6.43	5.59	-11.14	8.27	-1.74	11.60	912828EA4
Feb-06/Mar-10	50	5.58	6.74	6.60	-10.62	8.88	-1.18	9.10	912828ET3
Aug-06/Mar-10	44	6.43	7.98	6.95	-10.49	9.42	-1.07	8.23	912828FL9
Feb-07/Mar-10	38	7.01	10.07	7.33	-10.05	10.15	-0.81	6.70	912828GD6
Aug-07/Mar-10	32	6.85	10.04	6.96	-9.21	10.48	-0.61	5.41	912828GX2
Feb-08/Mar-10	26	2.69	7.99	6.53	-9.02	10.62	-0.75	4.93	912828HN3
Aug-08/Mar-10	20	3.06	9.04	6.71	-8.12	11.58	-0.47	3.65	912828JE1
Feb-09/Mar-10	14	5.32	7.07	6.41	-3.41	8.77	0.61	3.36	912828JX9
Aug-09/Mar-10	8	6.33	14.76	2.70	-2.84	6.83	-0.62	2.00	912828LA6
c) Bonds (19-year to 20-year and 29-year to 30-year)									
May-98/Mar-10	143	7.40	7.03	10.16	-11.30	10.35	-0.54	5.56	912810FD5
May-99/Mar-10	131	7.94	7.32	10.09	-11.33	10.85	-0.56	5.23	912810FH6
Nov-01/Mar-10	101	7.00	6.83	13.74	-10.91	13.44	-0.17	4.44	912810FQ6
Aug-04/Mar-10	68	5.58	4.61	8.08	-11.96	10.59	-0.70	6.22	912810FR4
Feb-06/Mar-10	50	4.23	3.88	10.15	-11.46	12.42	-0.26	4.99	912810FS2
Feb-07/Mar-10	38	6.06	8.23	10.04	-11.86	13.76	-0.36	4.58	912810PS1
Feb-08/Mar-10	26	0.48	1.19	9.19	-11.76	16.17	-0.13	3.48	912810PV4
Feb-09/Mar-10	14	8.01	14.20	8.92	-5.95	13.14	0.24	3.02	912810PZ5

Date	N	mean	med	monthly		std	skew	kurt	CUSIP
from/to				max	min				
II) U.S. Treasury Nominal Bonds									
a) Short-term notes (4-year to 5-year)									
Aug-97/Feb-02	55	6.12	5.62	2.64	-1.54	2.38	0.24	5.06	9128273C4
May-05/Mar-10	59	3.88	3.06	2.41	-1.50	2.44	0.33	4.25	912828DR8
Jun-06/Mar-10	46	5.95	4.95	3.16	-1.63	3.04	0.61	4.53	912828FD7
May-07/Mar-10	35	6.55	4.60	3.44	-1.95	4.03	0.46	3.52	912828GQ7
May-08/Mar-10	23	5.01	6.56	4.48	-1.72	4.67	0.83	4.77	912828HY9
May-09/Mar-10	11	1.30	3.74	1.81	-2.37	4.42	-0.49	2.38	912828KN9
b) Notes (9-year to 10-year)									
Mar-97/Jan-07	119	6.07	4.52	4.35	-3.73	4.42	0.07	4.24	9128272J0
Mar-98/Jan-08	119	5.36	4.66	4.62	-4.28	4.62	0.02	4.99	9128273X8
Dec-98/Oct-08	119	4.67	4.53	3.51	-4.48	4.74	-0.44	4.63	9128274V1
Mar-00/Jan-10	119	5.80	5.06	3.71	-4.37	4.81	-0.32	4.54	9128275Z1
Mar-01/Mar-10	109	5.04	4.25	4.19	-5.33	5.50	-0.46	4.71	9128276T4
Mar-02/Mar-10	97	5.43	5.20	4.42	-5.96	5.79	-0.56	5.19	9128277L0
Sep-02/Mar-10	91	4.88	5.01	4.70	-6.44	5.99	-0.51	5.60	912828AJ9
Sep-03/Mar-10	79	5.55	7.54	4.57	-4.94	5.52	-0.22	4.17	912828BH2
Mar-04/Mar-10	73	5.06	7.39	4.75	-5.03	5.69	-0.30	4.00	912828CA6
Sep-04/Mar-10	67	5.35	7.47	4.86	-2.87	5.49	0.07	2.96	912828CT5
Mar-05/Mar-10	61	5.85	7.43	6.07	-2.89	6.03	0.33	3.55	912828DM9
Sep-05/Mar-10	55	5.34	6.08	7.60	-3.14	6.86	0.73	4.71	912828EE6
Mar-06/Mar-10	49	6.43	7.62	7.93	-3.46	7.17	0.70	4.88	912828EW6
Sep-06/Mar-10	43	6.96	7.86	7.88	-3.48	7.43	0.73	4.67	912828FQ8
Mar-07/Mar-10	37	6.86	6.64	7.73	-3.91	8.26	0.55	3.93	912828GH7
Sep-07/Mar-10	31	7.04	8.35	7.61	-4.02	8.81	0.53	3.63	912828HA1
Mar-08/Mar-10	25	3.64	4.69	7.72	-4.83	9.77	0.55	3.78	912828HR4
Sep-08/Mar-10	19	4.68	2.68	8.75	-4.91	11.38	0.74	3.75	912828JH4
Mar-09/Mar-10	13	-1.80	2.79	3.28	-4.60	8.11	-0.54	2.38	912828KD1
Sep-09/Mar-10	7	-1.63	1.78	2.34	-4.73	8.16	-1.03	3.15	912828LJ7
c) Bonds (19-year to 20-year and 29-year to 30-year)									
Sep-98/Mar-10	139	5.78	9.26	13.58	-10.72	11.39	-0.21	5.43	912810FE3
Mar-99/Mar-10	133	6.23	11.02	13.66	-11.22	11.58	-0.20	5.68	912810FG8
Mar-01/Mar-10	109	5.87	11.43	14.01	-12.02	12.76	-0.18	5.42	912810FP8
Aug-04/Mar-10	68	6.21	8.12	12.63	-8.63	10.63	0.52	6.35	912810ET1
Feb-06/Mar-10	50	5.33	6.70	12.74	-8.62	11.82	0.57	5.98	912810EW4
Feb-07/Mar-10	38	6.11	5.43	12.78	-8.85	12.87	0.55	5.53	912810EZ7
Feb-08/Mar-10	26	3.37	3.61	12.83	-9.80	15.46	0.50	4.60	912810FB9
Feb-09/Mar-10	14	-2.44	2.40	5.60	-6.33	11.36	-0.42	2.79	912810FG8

Table 3: Nominal monthly log returns

A) Nominal monthly log returns for the complete period, March 1997 to March 2010. The mean, median, and standard deviation of monthly returns are annualized (in percentage terms). For each group of TIPS we construct an index return with the return of the on-the-run security. At any time within each TIPS group we obtain the most recent issued bond that will coincide with the newest issuance within the group. We obtain a comparable nominal bond by choosing the closest in maturity to our TIPS benchmark.

					monthly						
Financial Asset	*N*	**mean**	**median**	**avg(m, me)**	**max**	**min**	**std**	**skew**	**kurt**	**mean/std**	**med/std**
TIPS 5-year Index	152	6.09	6.04	6.06	3.83	-4.09	3.67	-0.554	5.810	1.661	1.648
TIPS 10-year Index	157	6.30	6.95	6.62	6.41	-8.12	6.25	-0.678	6.999	1.008	1.112
TIPS LT Bond Index	139	7.56	7.88	7.72	9.21	-11.76	11.11	-0.583	5.365	0.680	0.709
Nominal 5-year Index	152	5.52	5.68	5.60	4.48	-3.22	4.02	-0.054	4.398	1.373	1.414
Nominal 10-year Index	157	5.96	6.65	6.30	8.75	-6.44	7.52	-0.037	4.416	0.792	0.884
Nominal LT Bond Index	139	5.25	8.06	6.66	12.83	-11.11	11.26	-0.209	5.405	0.466	0.716
S&P 500 Index	157	4.73	12.92	8.83	9.33	-18.39	16.76	-0.844	4.225	0.282	0.771
Gold	157	8.81	2.28	5.55	16.01	-12.48	13.81	0.266	4.530	0.638	0.165
Commodity Index	157	2.44	6.83	4.64	17.95	-33.13	24.74	-0.683	5.139	0.099	0.276
Real Estate Index	155	5.46	9.01	7.24	2.27	-2.83	3.75	-1.165	4.033	1.458	2.407
Inflation (CPI-U NSA)	157	2.36	2.36	2.36	1.21	-1.93	1.36	-1.228	8.376	1.734	1.729

B) Nominal monthly log returns for the entire business cycle (peak to peak) according to the Business Cycle Committee of the National Bureau of Economic Research, March 2001 to December 2007.

					monthly						
Financial Asset	*N*	**mean**	**median**	**avg(m, me)**	**max**	**min**	**std**	**skew**	**kurt**	**mean/std**	**med/std**
TIPS 5-year Index	81	6.30	5.58	5.94	3.83	-2.68	3.69	-0.052	4.275	1.706	1.512
TIPS 10-year Index	81	7.16	8.52	7.84	4.88	-4.80	5.84	-0.434	4.038	1.225	1.458
TIPS LT Bond Index	81	9.02	8.21	8.62	9.21	-10.22	10.80	-0.698	4.889	0.835	0.760
Nominal 5-year Index	81	5.09	5.00	5.05	3.44	-3.22	4.16	-0.280	3.985	1.222	1.202
Nominal 10-year Index	81	5.44	7.61	6.52	4.70	-6.44	7.38	-0.501	3.532	0.737	1.031
Nominal LT Bond Index	81	6.57	11.42	9.00	8.56	-11.11	10.71	-0.718	4.792	0.614	1.066
S&P 500 Index	81	5.24	11.59	8.41	8.44	-11.51	12.78	-0.587	4.009	0.410	0.907
Gold	81	18.19	14.16	16.18	10.22	-12.48	13.19	-0.246	4.326	1.379	1.074
Commodity Index	81	10.32	13.87	12.09	14.10	-15.56	21.68	-0.324	2.675	0.476	0.640
Real Estate Index	81	8.03	10.13	9.08	2.27	-2.26	3.15	-0.889	4.003	2.551	3.218
Inflation (CPI-U NSA)	81	2.62	2.52	2.57	1.21	-0.81	1.30	-0.159	2.890	2.016	1.941

C) Nominal monthly log returns for the two recessionary periods where TIPS were available, March 2001 to November 2001, and December 2007 to June 2009.

					monthly						
Financial Asset	*N*	**mean**	**median**	**avg(m, me)**	**max**	**min**	**std**	**skew**	**kurt**	**mean/std**	**med/std**
TIPS 5-year Index	28	3.86	4.80	4.33	2.90	-4.10	5.36	-0.669	3.903	0.719	0.895
TIPS 10-year Index	28	2.96	7.20	5.08	6.40	-8.10	10.19	-0.470	4.215	0.290	0.706
TIPS LT Bond Index	28	2.74	-0.60	1.07	9.00	-11.80	15.67	-0.221	3.682	0.175	-0.038
Nominal 5-year Index	28	6.04	7.20	6.62	4.50	-2.00	4.77	0.641	4.317	1.267	1.510
Nominal 10-year Index	28	5.70	3.00	4.35	8.70	-4.90	10.37	0.580	3.577	0.550	0.289
Nominal LT Bond Index	28	5.53	2.40	3.96	12.80	-9.80	16.32	0.449	3.774	0.339	0.147
Equity	28	-21.90	-9.00	-15.45	9.10	-18.40	23.17	-0.308	2.702	-0.945	-0.388
Gold	28	8.57	-1.20	3.69	10.20	-11.40	16.74	-0.137	3.108	0.512	-0.072
Commodity	28	-34.63	-24.60	-29.61	18.00	-33.10	35.65	-0.591	4.102	-0.971	-0.690
Real Estate	28	-9.51	-12.00	-10.76	1.40	-2.80	4.96	0.124	1.405	-1.920	-2.421
Inflation (CPI-U NSA)	28	1.20	3.00	2.10	1.00	-1.90	2.24	-1.185	4.595	0.535	1.337

Table 4: Correlations matrix of nominal monthly log returns and other asset classes

A) Period March 1997 to March 2010.

		Inflation	TIPS			Nominal			S&P 500	Gold	S&P GSCI	S&P CS10
			5-year	10-year	LT Bond	5-year	10-year	LT Bond				
		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
(1)	**Inflation (CPI-U NSA)**	1.000	0.220	0.174	0.094	-0.239	-0.143	-0.109	0.168	0.182	0.695	0.084
(2)	**TIPS 5-year Index**	0.220	1.000	0.920	0.812	0.669	0.672	0.616	-0.050	0.159	0.311	-0.008
(3)	**TIPS 10-year Index**	0.174	0.920	1.000	0.895	0.645	0.738	0.706	0.021	0.107	0.251	0.013
(4)	**TIPS LT Bond Index**	0.094	0.812	0.895	1.000	0.583	0.739	0.787	0.051	0.034	0.207	0.050
(5)	**Nominal 5-year Index**	-0.239	0.669	0.645	0.583	1.000	0.874	0.746	-0.286	0.016	-0.127	-0.147
(6)	**Nominal 10-year Index**	-0.143	0.672	0.738	0.739	0.874	1.000	0.943	-0.211	-0.015	-0.046	-0.081
(7)	**Nominal LT Bond Index**	-0.109	0.616	0.706	0.787	0.746	0.943	1.000	-0.124	-0.024	-0.003	-0.041
(8)	**S&P 500 Index**	0.168	-0.050	0.021	0.051	-0.286	-0.211	-0.124	1.000	0.031	0.212	0.183
(9)	**Gold**	0.182	0.159	0.107	0.034	0.016	-0.015	-0.024	0.031	1.000	0.257	-0.071
(10)	**Commodity Index**	0.695	0.311	0.251	0.207	-0.127	-0.046	-0.003	0.212	0.257	1.000	0.254
(11)	**Real Estate Index**	0.084	-0.008	0.013	0.050	-0.147	-0.081	-0.041	0.183	-0.071	0.254	1.000

B) Sample data of an entire business cycle (peak from previous peak) from March 2001 to December 2007, according to the Business Cycle Committee of the National Bureau of Economic Research.

		Inflation	TIPS			Nominal			S&P 500	Gold	S&P GSCI	S&P CS10
			5-year	10-year	LT Bond	5-year	10-year	LT Bond				
		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
(1)	**CPI-U NSA**	1.000	0.065	0.032	-0.022	-0.111	-0.071	-0.063	-0.099	0.172	0.645	-0.037
(2)	**TIPS 5-year Index**	0.065	1.000	0.932	0.813	0.838	0.814	0.729	-0.384	-0.024	0.219	-0.131
(3)	**TIPS 10-year Index**	0.032	0.932	1.000	0.910	0.790	0.871	0.834	-0.340	-0.073	0.204	-0.042
(4)	**TIPS Bond Index**	-0.022	0.813	0.910	1.000	0.688	0.808	0.851	-0.199	-0.066	0.202	-0.009
(5)	**Nominal 5-year Index**	-0.111	0.838	0.790	0.688	1.000	0.876	0.756	-0.405	-0.054	-0.039	-0.141
(6)	**Nominal 10-year Index**	-0.071	0.814	0.871	0.808	0.876	1.000	0.946	-0.480	-0.119	0.004	-0.064
(7)	**Nominal Bond Index**	-0.063	0.729	0.834	0.851	0.756	0.946	1.000	-0.350	-0.105	0.036	-0.038
(8)	**S&P 500 Index**	-0.099	-0.384	-0.340	-0.199	-0.405	-0.480	-0.350	1.000	0.200	-0.068	-0.077
(9)	**Gold**	0.172	-0.024	-0.073	-0.066	-0.054	-0.119	-0.105	0.200	1.000	0.260	-0.182
(10)	**Commodity Index**	0.645	0.219	0.204	0.202	-0.039	0.004	0.036	-0.068	0.260	1.000	0.048
(11)	**Real Estate Index**	-0.037	-0.131	-0.042	-0.009	-0.141	-0.064	-0.038	-0.077	-0.182	0.048	1.000

Table 5: Results for short-term investors

Optimal portfolio weights for short-term investors who are able (not able) to buy TIPS. Optimal weights are computed for different levels of relative risk aversion and combinations of asset classes. The benefits provided by TIPS are computed in basis points (real terms) as $\mathrm{E}_t\left[r_{p,t+1}^{(tips)}\right] - \frac{\sigma_{r_p^{(tips)}}}{\sigma_{r_p}}\mathrm{E}_t\left[r_{p,t+1}\right]$.

	Optimal Weights in %											
	$\gamma=\infty$		$\gamma=20$		$\gamma=10$		$\gamma=5$		$\gamma=3$		$\gamma=1$	
i) Stocks, 10-year nominal bonds, 10-year TIPS and T-bills												
Stocks	1.3	(1.4)	13.2	(15.8)	24.7	(30.5)	35.7	(41.1)	48.8	(52.1)	100	(100)
10-year nominal bonds	0	(0)	6.6	(30.5)	20.6	(63.3)	22.9	(59)	25.8	(47.9)	0	(0)
10-year TIPS	3.7	(0)	37	(0)	54.7	(0)	41.4	(0)	25.4	(0)	0	(0)
T-bills	94.9	(98.6)	43.2	(53.7)	0	(6.2)	0	(0)	0	(0)	0	(0)
$\mathrm{E}_t\left[r_{p,t+1}^{(tips)}\right] - \frac{\sigma_{r_p^{(tips)}}}{\sigma_{r_p}}\mathrm{E}_t\left[r_{p,t+1}\right]$	14bp		20bp		35bp		31bp		16bp		0bp	
ii) Stocks, 10-year nominal bonds, 10-year TIPS, Gold and T-bills												
Stocks	1.3	(1.3)	13.3	(15.6)	23.7	(28.5)	35	(38.7)	48.6	(51)	100	(100)
10-year nominal bonds	0	(0)	9.4	(30.6)	22.1	(57.3)	24	(51.4)	26.2	(44.3)	0	(0)
10-year TIPS	3.4	(0)	32.9	(0)	44	(0)	34.2	(0)	22.5	(0)	0	(0)
Gold	1.6	(1.8)	7.1	(8.7)	10.2	(14.2)	6.8	(9.9)	2.7	(4.7)	0	(0)
T-bills	93.8	(96.9)	37.4	(45.1)	0	(0)	0	(0)	0	(0)	0	(0)
$\mathrm{E}_t\left[r_{p,t+1}^{(tips)}\right] - \frac{\sigma_{r_p^{(tips)}}}{\sigma_{r_p}}\mathrm{E}_t\left[r_{p,t+1}\right]$	12bp		16bp		29bp		21bp		11bp		0bp	
iii) Stocks, 10-year nominal bonds, 10-year TIPS, Commodities and T-bills												
Stocks	0.2	(0.2)	12.7	(14.1)	24.2	(28.4)	35.2	(38.5)	48.4	(50.3)	100	(100)
10-year nominal bonds	0	(0)	11.7	(30.5)	25.5	(63.4)	27.3	(56)	29.5	(45.7)	0	(0)
10-year TIPS	0	(0)	29.1	(0)	47	(0)	34.5	(0)	19.5	(0)	0	(0)
Commodities	3.8	(3.8)	3.4	(5.3)	3.3	(6.8)	2.9	(5.6)	2.5	(4)	0	(0)
T-bills	96	(96)	43.1	(50.1)	0	(1.5)	0	(0)	0	(0)	0	(0)
$\mathrm{E}_t\left[r_{p,t+1}^{(tips)}\right] - \frac{\sigma_{r_p^{(tips)}}}{\sigma_{r_p}}\mathrm{E}_t\left[r_{p,t+1}\right]$	0bp		8bp		26bp		20bp		9bp		0bp	
iv) Stocks, 10-year nominal bonds, 10-year TIPS, Real estate and T-bills												
Stocks	1.3	(1.3)	7.5	(8.8)	13	(13.9)	24	(24)	36.2	(36.2)	100	(100)
10-year nominal bonds	0	(0)	3.7	(17.1)	5	(13.9)	7.4	(7.4)	0	(0)	0	(0)
10-year TIPS	3.7	(0)	19.2	(0)	12.8	(0)	0	(0)	0	(0)	0	(0)
Real estate	1.9	(2)	69.6	(74.1)	69.3	(72.2)	68.6	(68.6)	63.8	(63.8)	0	(0)
T-bills	93.1	(96.7)	0	(0)	0	(0)	0	(0)	0	(0)	0	(0)
$\mathrm{E}_t\left[r_{p,t+1}^{(tips)}\right] - \frac{\sigma_{r_p^{(tips)}}}{\sigma_{r_p}}\mathrm{E}_t\left[r_{p,t+1}\right]$	13bp		17bp		12bp		0bp		0bp		0bp	
v) All Assets												
Stocks	0.2	(0.2)	7.3	(8.2)	12.9	(13.4)	24	(24)	36.2	(36.2)	100	(100)
10-year nominal bonds	0	(0)	5.5	(15.8)	6.4	(13.1)	7.5	(7.5)	0	(0)	0	(0)
10-year TIPS	0	(0)	14.8	(0)	9.7	(0)	0	(0)	0	(0)	0	(0)
Gold	0	(0)	4.9	(5.5)	3.1	(3.5)	0	(0)	0	(0)	0	(0)
Commodities	3.8	(3.8)	0.3	(1.2)	0.4	(0.9)	0.3	(0.3)	0	(0)	0	(0)
Real estate	0	(0)	67.1	(69.4)	67.6	(69.1)	68.3	(68.3)	63.8	(63.8)	0	(0)
T-bills	96	(96)	0	(0)	0	(0)	0	(0)	0	(0)	0	(0)
$\mathrm{E}_t\left[r_{p,t+1}^{(tips)}\right] - \frac{\sigma_{r_p^{(tips)}}}{\sigma_{r_p}}\mathrm{E}_t\left[r_{p,t+1}\right]$	0bp		10bp		6bp		0bp		0bp		0bp	

Table 6: Results for long-term investors

Optimal portfolio weights for long-term investors who are able (not able) to buy TIPS. Optimal weights are computed for different levels of relative risk aversion and combinations of asset classes. The benefits provided by TIPS are computed in basis points (real terms) as $\mathrm{E}_t\left[r_{p,t+1}^{(tips)}\right] - \frac{\sigma_{r_p^{(tips)}}}{\sigma_{r_p}}\mathrm{E}_t\left[r_{p,t+1}\right]$.

	Optimal Weights in %											
	$\gamma=\infty$		$\gamma=20$		$\gamma=10$		$\gamma=5$		$\gamma=3$		$\gamma=1$	
i) Stocks, 10-year nominal bonds and 10-year TIPS												
Stocks	0	(1.4)	9.7	(11.4)	19.4	(21.4)	39	(41.6)	62.5	(65.9)	100	(100)
10-year nominal bonds	0	(98.6)	0	(88.6)	0	(78.6)	0	(58.4)	0	(34.1)	0	(0)
10-year TIPS	100	(0)	90.3	(0)	80.6	(0)	61	(0)	37.6	(0)	0	(0)
$\mathrm{E}_t\left[r_{p,t+1}^{(tips)}\right] - \frac{\sigma_{r_p^{(tips)}}}{\sigma_{r_p}}\mathrm{E}_t\left[r_{p,t+1}\right]$	248bp		89bp		50bp		31bp		21bp		0bp	
ii) Stocks, Gold, 10-year nominal bonds and 10-year TIPS												
Stocks	0	(1.3)	9.6	(11.2)	19.4	(21.2)	38.9	(41.1)	62.3	(65.1)	100	(100)
Gold	0	(1.8)	4.3	(6.4)	8.6	(11.1)	17.2	(20.5)	27.6	(31.8)	0	(0)
10-year nominal bonds	0	(96.9)	0	(82.4)	0	(67.8)	0	(38.4)	0	(3.2)	0	(0)
10-year TIPS	100	(0)	86.1	(0)	72.1	(0)	43.9	(0)	10.2	(0)	0	(0)
$\mathrm{E}_t\left[r_{p,t+1}^{(tips)}\right] - \frac{\sigma_{r_p^{(tips)}}}{\sigma_{r_p}}\mathrm{E}_t\left[r_{p,t+1}\right]$	248bp		82bp		45bp		25bp		15bp		0bp	
iii) Stocks, Commodities, 10-year nominal bonds and 10-year TIPS												
Stocks	0	(0.2)	9.4	(9.9)	18.9	(19.6)	37.8	(39.2)	60.6	(62.7)	100	(100)
Commodities	0	(3.8)	1.3	(4.8)	2.6	(5.8)	5.1	(7.8)	8.2	(10.1)	0	(0)
10-year nominal bonds	0	(96)	0	(85.4)	0	(74.6)	0	(53)	0	(27.1)	0	(0)
10-year TIPS	100	(0)	89.4	(0)	78.6	(0)	57.1	(0)	31.2	(0)	0	(0)
$\mathrm{E}_t\left[r_{p,t+1}^{(tips)}\right] - \frac{\sigma_{r_p^{(tips)}}}{\sigma_{r_p}}\mathrm{E}_t\left[r_{p,t+1}\right]$	248bp		61bp		35bp		22bp		15bp		0bp	
iv) Stocks, Real estate, 10-year nominal bonds and 10-year TIPS												
Stocks	0	(1.3)	7.1	(7.5)	12.8	(12.8)	23.4	(23.4)	36.2	(36.2)	100	(100)
Real estate	0	(2)	77.2	(92.5)	87.2	(87.2)	76.6	(76.6)	63.8	(63.8)	0	(0)
10-year nominal bonds	0	(96.7)	0	(0)	0	(0)	0	(0)	0	(0)	0	(0)
10-year TIPS	100	(0)	15.7	(0)	0	(0)	0	(0)	0	(0)	0	(0)
$\mathrm{E}_t\left[r_{p,t+1}^{(tips)}\right] - \frac{\sigma_{r_p^{(tips)}}}{\sigma_{r_p}}\mathrm{E}_t\left[r_{p,t+1}\right]$	248bp		37bp		0bp		0bp		0bp		0bp	
v) All Assets												
Stocks	0	(0.2)	6.9	(7)	12.4	(12.4)	23.4	(23.4)	36.2	(36.2)	100	(100)
Gold	0	(0)	6.5	(7.1)	4.9	(4.9)	0.3	(0.3)	0	(0)	0	(0)
Commodities	0	(3.8)	0.6	(0.7)	0.5	(0.5)	0.1	(0.1)	0	(0)	0	(0)
Real estate	0	(0)	79.8	(85.3)	82.2	(82.2)	76.2	(76.2)	63.8	(63.8)	0	(0)
10-year nominal bonds	0	(96)	0	(0)	0	(0)	0	(0)	0	(0)	0	(0)
10-year TIPS	100	(0)	6.2	(0)	0	(0)	0	(0)	0	(0)	0	(0)
$\mathrm{E}_t\left[r_{p,t+1}^{(tips)}\right] - \frac{\sigma_{r_p^{(tips)}}}{\sigma_{r_p}}\mathrm{E}_t\left[r_{p,t+1}\right]$	248bp		15bp		0bp		0bp		0bp		0bp	

Figure 1: Time series of TIPS available for investors

We group available TIPS according to their term to maturity. The red area corresponds to TIPS with less than 1-year to maturity; the grey area corresponds to TIPS between 1-year and 5-years to maturity; the blue area corresponds to TIPS between 5-years and 10-years to maturity; and, the brown area corresponds to TIPS with more than 10-years to maturity.

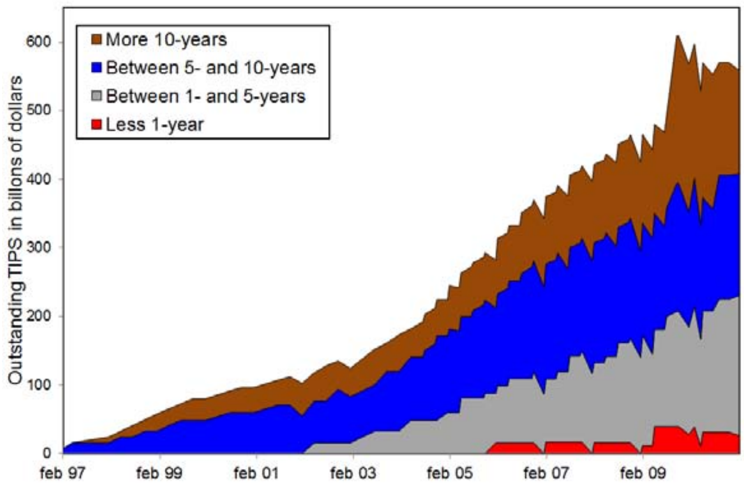

Figure 2: Nominal and real yields

Nominal and real yields are 10-year constant maturity from GSW (2010). The 10-Year-Ahead Inflation Forecasts is from the Survey of Professional Forecasters.

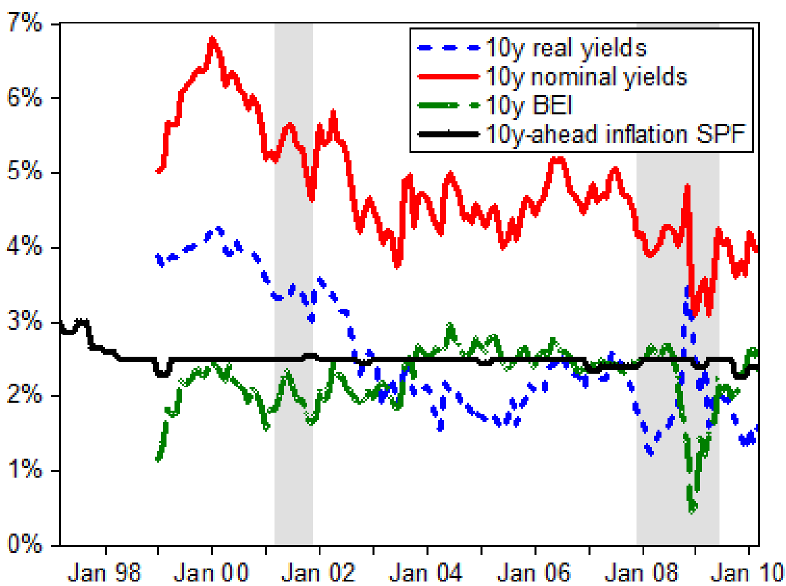

Figure 3: Annualized standard deviation of returns

Standard deviation of returns are computed using a 3-year rolling estimation.

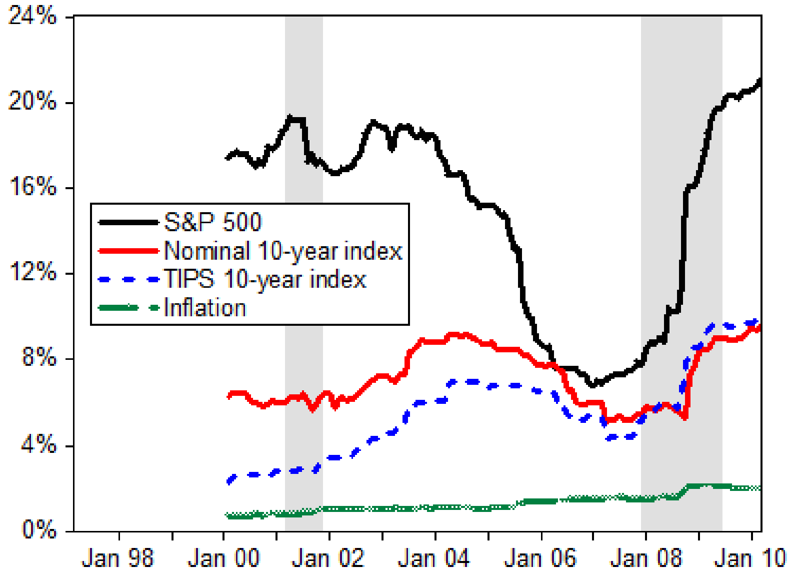

Figure 4: Conditional beta 3-year rolling window

The beta is calculated as the covariance between the bonds' returns and the S&P 500 returns divided by the variance of the S&P 500 returns.

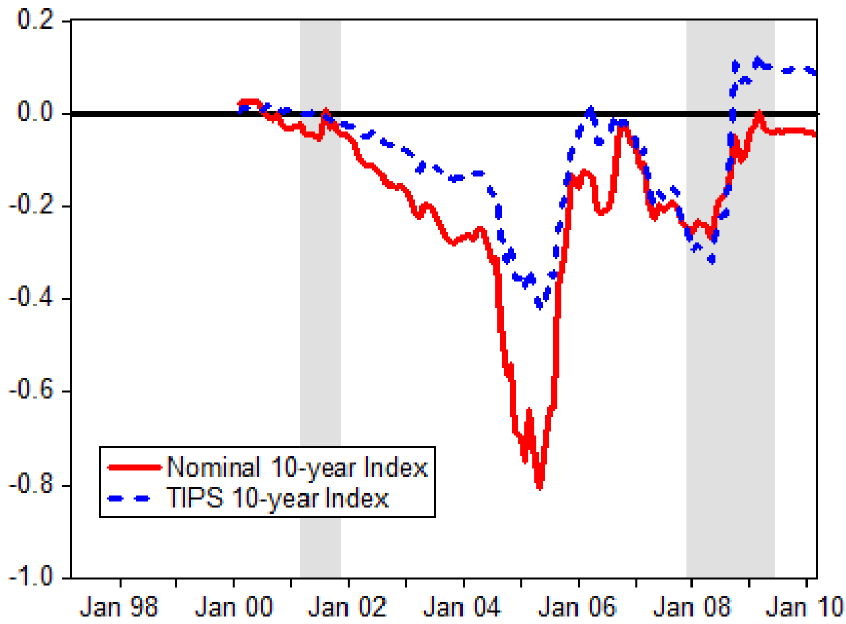

Figure 5: Optimal portfolio weights for a short-term investor

Optimal portfolio choice when a short-term investor is able to allocate his wealth into stocks, nominal bonds, and cash.

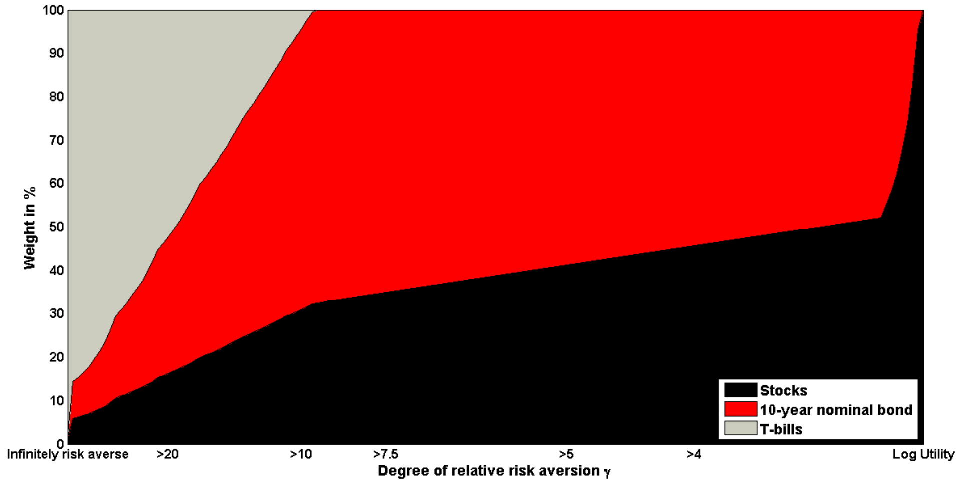

Figure 6: Optimal portfolio weights for a short-term investor

Optimal portfolio choice when a short-term investor is able to allocate his wealth into stocks, nominal bonds, TIPS and cash.

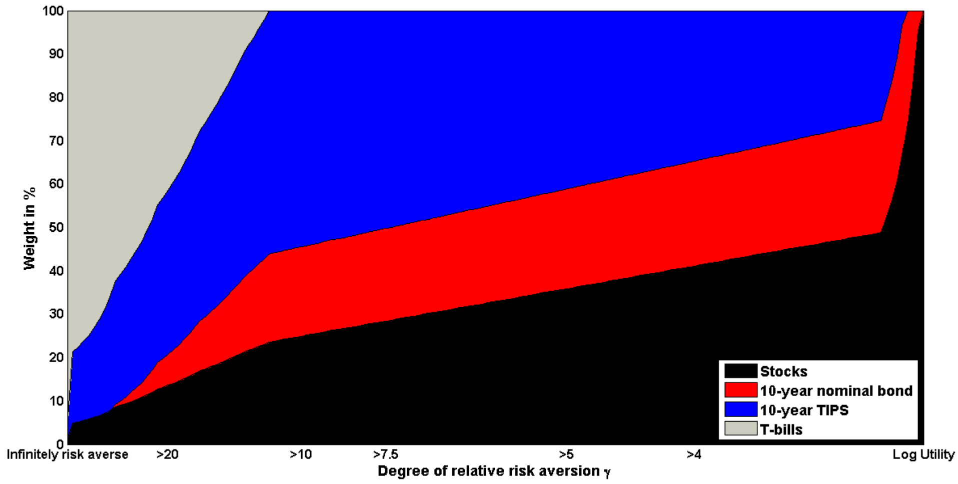

Figure 7: Optimal portfolio weights for a long-term investor

Optimal portfolio choice when a long-term investor is able to allocate her wealth into stocks, and the nominal long-term risk-free asset

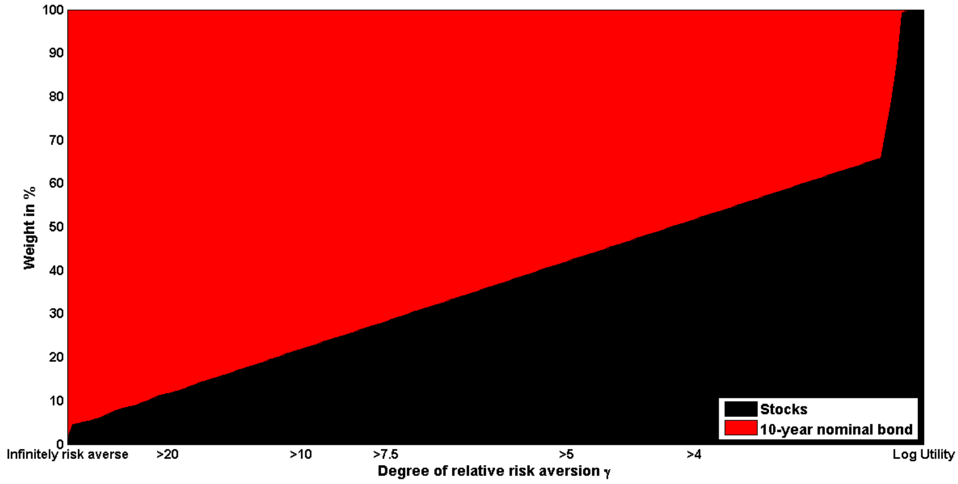

Figure 8: Optimal portfolio weights for a long-term investor

Optimal portfolio choice when a long-term investor is able to allocate her wealth into stocks, the nominal long-term risk-free asset and TIPS which represents the real long-term risk-free asset.